Homework 2
Section 2.3

12. Adjoin the identity matrix form

$$[A \,\vdots\, I] = \begin{bmatrix} 10 & 5 & -7 & \vdots & 1 & 0 & 0 \\ -5 & 1 & 4 & \vdots & 0 & 1 & 0 \\ 3 & 2 & -2 & \vdots & 0 & 0 & 1 \end{bmatrix}.$$

Using elementary row operations, reduce the matrix to

$$[I \,\vdots\, A^{-1}] = \begin{bmatrix} 1 & 0 & 0 & \vdots & -10 & -4 & 27 \\ 0 & 1 & 0 & \vdots & 2 & 1 & -5 \\ 0 & 0 & 1 & \vdots & -13 & -5 & 35 \end{bmatrix}$$

Therefore, the inverse is

$$A^{-1} = \begin{bmatrix} -10 & -4 & 27 \\ 2 & 1 & -5 \\ -13 & -5 & 35 \end{bmatrix}.$$

38. The inverse of A is given by

$$A^{-1} = \frac{1}{x-4}\begin{bmatrix} -2 & -x \\ 1 & 2 \end{bmatrix}.$$

Letting $A^{-1} = A$, you find that $\dfrac{1}{x-4} = -1$.

So, $x = 3$.

46. (a) True. If A_1, A_2, A_3, A_4 are invertible 7×7 matrices, then $B = A_1A_2A_3A_4$ is also an invertible 7×7 matrix with inverse $B^{-1} = A_4^{-1}A_3^{-1}A_2^{-1}A_1^{-1}$, by Theorem 2.9 on page 81 and induction.

(b) True. $\left(A^{-1}\right)^T = \left(A^T\right)^{-1}$ by Theorem 2.8(4) on page 79.

(c) False. For example consider the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ which is not invertible, but $1 \cdot 1 - 0 \cdot 0 = 1 \neq 0$.

(d) False. If A is a square matrix then the system $A\mathbf{x} = \mathbf{b}$ has a unique solution if and only if A is a *nonsingular* matrix.

54. Prove that if $A^2 = A$ then either A = I or A is singular.

Proof:

Let A be a matrix such that $A^2 = A$. We know that any matrix is singular or non-singular. If A is singular the conclusion is true. Suppose A is non-singular. In this case A has an inverse. Multiplying both sides of $A^2 = A$ by A^{-1}, we see $A^{-1}A^2 = A^{-1}A$. Simplifying both sides of this last equation yields A = I. Thus A is either singular or A = I. QED.

<u>Section 2.4</u>

10. C is obtained by adding the third row of A to the first row. So,

$$E = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

30. Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{bmatrix}$ as follows.

Elementary Row Operation	Elementary Matrix	Resulting Matrix
-2 times row one to row two	$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 3 & 4 \end{bmatrix}$
-1 times row one to row three	$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$
-1 times row two to row three	$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
-3 times row three to row one	$E_4 = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
-2 times row two to row one	$E_5 = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

So, one way to factor A is $A = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}E_5^{-1} =$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

34. (a) False. It is impossible to obtain the zero matrix by applying any elementary row operation to the identity matrix.

(b) True. See the definition of row equivalence on page 90.

(c) True. If $A = E_1 E_2 \ldots E_k$, where each E_i is an elementary matrix, then A is invertible (because every elementary matrix is) and $A^{-1} = E_k^{-1} \ldots E_2^{-1} E_1^{-1}$.

(d) True. See equivalent conditions (2) and (3) of Theorem 2.15 on page 93.

Section 3.1

28. Expand along the third row (which is all zeros). The determinant is clearly zero because you add up a bunch of zeros.

48. (a) False. The determinant of a triangular matrix is equal to the *product* of the entries on the main diagonal. For example, if

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \text{ then } \det(A) = 2 \neq 3 = 1 + 2.$$

(b) True. See Theorem 3.1 on page 126.

(c) True. This is because in a cofactor expansion each cofactor gets multiplied by the corresponding entry. If this entry is zero, the product would be zero independent of the value of the cofactor.